

MULTIPLES SUPPRESSION USING NON LINEAR FILTERING IN (τ, p) DOMAIN

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ABSTRACT

In seismic processing, the suppression of multiples is often considered as an important step. Initially the conventional filter for suppression of multiples in the (f, k) domain has proved efficient namely for the far offsets. However, due to some arising geological difficulties (especially when the multiple energy is superposes to that of the primary reflection) this filter is no more appropriate.

New techniques are then required. Non linear filtering allows us to define automatically the zone to eliminate, even energies of events (multiple and primary reflections) are superposed or are overlapped.

To release this filter, the Butterworth gain function was used to compare the energies of the primaries and multiples reflections. The non linear filter is then applied using the original model and the predicted multiple model. This latter leads to better results.

This was the subject of a first work.

Filtering by the linear τ - p transformation has also given good results. However, the parabolic transformation gives much better results. Its main advantage is that a reflection in the (x, t) domain, transforms into a point in the (τ, p) domain making its elimination much easier to perform when the reflection is a multiple.

In this work, we deal with the non linear filter using the predicted multiple model in (τ, p) domain (linear and parabolic).

Key words - Multiple suppression - Offsets - (f, k) and (τ, p) domains - τ - p transform - Linear-Parabolic - Non linear filtering - Butterworth filter - Predicted multiples model.

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SUPPRESSION DES MULTIPLES UTILISANT LE FILTRAGE NON LINÉAIRE DANS LE DOMAINE (τ, p)

RÉSUMÉ

En traitement sismique, la suppression des réflexions multiples est souvent répertoriée comme une étape importante. Initialement, le filtrage conventionnel pour la suppression des multiples dans le domaine (f, k) a été efficace, particulièrement pour les offsets lointains. Cependant, ce filtre n'est plus approprié dans le cas où les réflexions simples et multiples se superposent.

Partant de la différence des vitesses apparentes des réflexions primaires et multiples, les limites du filtre conventionnel apparaissent vite lorsqu'il y a une superposition des énergies de ces réflexions.

Le filtre non linéaire pallie à cette difficulté. Il permet de définir automatiquement la zone du domaine (f, k) à rejeter, plutôt que de remettre à zéro (brutalement) la partie non désirée du spectre. Sur le modèle de données original (réflexions primaires et multiples) et sur celui des multiples (prédit par équation d'onde) a été appliqué ce filtre dans un premier travail. La fonction utilisée pour comparer les énergies est la fonction de gain de Butterworth. L'application du filtre non linéaire donne de meilleurs résultats en utilisant le modèle multiple prédit.

Dans le présent travail, nous utilisons le filtrage non linéaire avec la fonction de gain de Butterworth d'abord avec la transformée τ - p linéaire, puis la transformée parabolique sur le modèle multiple prédit.

Mots clés - Suppression - Multiples - Offsets - domaines (f, k) et (τ, p) - Transformation τ - p -Linéaire-Parabolique - Filtrage non linéaire - Fonction de gain de Butterworth - Modèle de multiple prédit.

1 - INTRODUCTION

The seismic trace is the result of a great number of waves provoked by a source, as well as noise of various origins. Some of these waves present a very strong coherence in the (x, t) domain such as a simple reflection or a good quality multiple. Others, on the contrary, as the natural noise, are practically random from one trace to another.

The detection of the useful signal and its extraction from a noised recording one is the ultimate objective of the seismic reflection (Yilmaz Özdoğan, 1993; 1987). Such extraction is possible only if the characteristics of both the noise and the signal are distinct. Some differences in terms of nature are evident namely those of parasites of low frequencies (Ground Roll). Others are finer, such as the multiple which is similar in characteristics to the simple

reflection. In this last case, the elimination of such parasites is more difficult.

The presence of multiples in a seismic section is one of the main obstacles for the identification of events; in fact, they can be easily confused with simple reflections, as they can contaminate or mask them. This is a common case in marine seismic (Taner, M.T., 1980).

Then the idea is to separate the reflections from multiples in order to eliminate these last (Arthur, B. and Weglein, B., 1999). This is possible via the representation of recordings in the (f, k) domain as well as the (τ, p) domain. Several methods are used for this purpose, each one with its specific techniques.

The determination of the zone to be eliminated (zone of multiples) is an importance task in terms of multiple attenuation in the (τ, p) domain. This

determination is fairly difficult - when one uses the conventional f-k filter - especially when the multiple is superposed on the primary reflection, or when the layers are inclined. In such a case either part of energy of the multiple is preserved, or the energy of the primary reflection is distorted. In addition, the conventional filter will give a poor impulsion response induced by the Gibbs phenomenon. An oval form of the zone to be eliminated is conditional to a good impulsion response; a form which is difficult to achieve.

A new filtering approach has been elaborated by Zhou and Greenhalgh (1994) for the suppression of multiples. It is a non linear filter applied on the predicted multiple records as well as on the original model using the Butterworth functions of gain. The main advantage of this filter closely relates to the good and automatic definition of the zone to eliminate. Furthermore, the smoothing of this zone promotes the reduction of the Gibbs phenomenon. This was shown in a previous work on multiples suppression in (f, k) domain.

The objective of this work is the study of filters for the suppression of multiples in (τ, p) domain. Our study focuses mainly on the conventional filter and the non linear filter with a Butterworth function of gain. The mathematical tools used here are the linear and parabolic τ - p transformations.

Applications are performed on three synthetic models.

2 - METHODS FOR MULTIPLE ATTENUATION

Multiples are energies arrivals that have undergone several reflections during their path. It is a phenomenon which occurs each time a contrast of acoustic impedance between layers takes place. Multiples are of two types : short or long (R.E. Sheriff and L.P. Geldart, 1983; I.P. Fail and G. Grau, 1992)

Short multiples are reflections that take place inside thin layers; they are also called reverberations. They affect the form of the wavelet by adding it

secondary lobes, decreasing thus the resolution. They are eliminated by the predictive deconvolution (Binzhong Zhou and Stewart Greenhalgh, 1994) (fig. 1-a).

Long multiples are reflections produced inside thick layers; they are particularly frequent in marine seismic, in the slice of water. This type of multiples is of great interest in the present work (fig. 1-b).

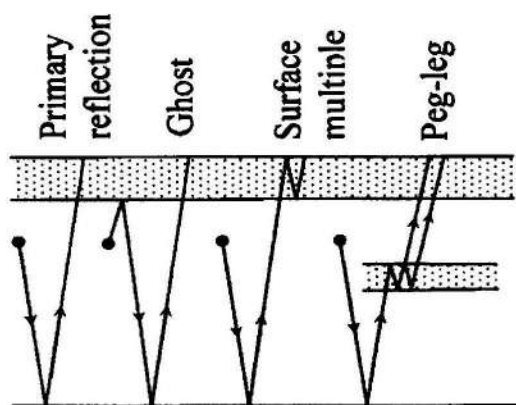


Fig. 1-a: Short Multiples

Fig. 1-a - Short Multiples
Multiples courts

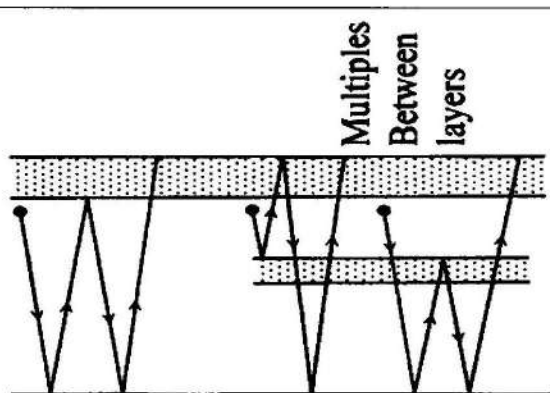


Fig. 1-b: Long Multiples

fig. 1-b - Long Multiples
Multiples longs

Distinguishing multiples from primary reflections involves three factors:

- The multiple can be detected by the time of its arrival in normal incidence.
- The multiple corrects dynamically by a lower velocity.
- The curvature of the hyperbole of the multiple is much greater.

3 - PRINCIPLE OF FILTERING

3-1 - Method of wave extrapolation

These last years, it became interesting to use the extrapolation method for the suppression of multiples (Wendell Winggins, 1999). This method can be considered as an extension of the predictive deconvolution method. It is based on the predicted multiple model, deduced from the original data. According to B. Zhou and S. Greenhalgh (1991), this extrapolation of the wave requires the use of very complex equations in addition to a considerable time calculation. The concise estimation of coefficients reflection constitutes the main difficulty when using this method.

3-2 - Non linear filter using the original record

This filter functions operates with the multiple model extracted from the original record which contains solely the undesirable part (the zone of multiples). By comparing the energy of the two records, the non linear filter will preserve automatically the useful part (negative area that contains the useful reflection energy) (R. Sellam and *al.*, 2005). The advantage of this filter is the automatic smoothing, instead of using ramps for each sample. But this method will not solve the problem of superposition of the multiple with the simple reflection or that of inclined layers.

4 - LINEAR AND PARABOLIC TRANSFORMATIONS

The conventional filter in the (f, k) domain proves its efficiency only for the large offsets. Its limitations become obvious when the energy of the multiple is superposed on that of the simple reflection.

Fairly better results are obtained by other techniques developed in the (τ, p) domain. In this work, we deal mainly with the contribution of the linear and parabolic τ - p transforms for the suppression of multiples.

The linear and parabolic τ - p transforms constitute the suitable mathematical tool for the suppression of multiples in the (τ, p) domain. Their principle consists in the summation of samples of a seismic surface according to a well determined direction (line or parable).

4-1 - Linear transform

The linear τ - p transform is a particular case of Radon transform. It consists in the summation of samples of a seismic recording along a line $t = \tau + px$, where τ denotes the intercept and p the ray parameter. Summing events according to a line of slope is merely their decomposition into plane waves.

There exists two variations of the linear τ - p transform. The first one consists in calculating the direct τ - p transform, then applying a filter on the inverse τ - p transform, whereas the second uses the filtering with the direct transform, the inverse being made simply by the inverse transform. In our case it is preferable to use the second approach to have a good resolution and therefore a much better separation of events in this area.

4-2 - The principle of the linear transform

Let's define in the (x, t) domain a set of lines under the form $t = \tau + px$ where p is the slope and τ the fixed intercept (fig 2) and a given function $s(t)$.

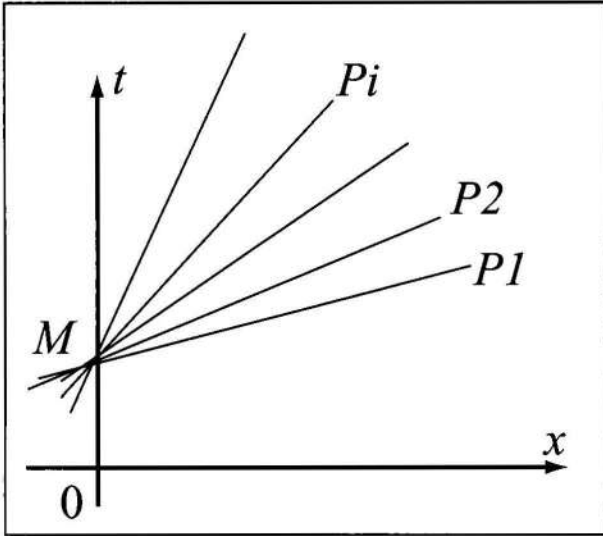


Fig. 2 - The bundle of lines $t = \tau + px$ used in the slant stack

Faisceau de droites appliqué dans l'addition oblique

The calculation of the integral $\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} s(x, t) dx dt$ regarding to this bundle of lines defines the linear τ - p transform $s_l(\tau, p)$ of $s(x, t)$ and which is expressed as

$$s_l(\tau, p) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} s(x, t) \delta[t - (\tau + px)] dt dx \quad (1)$$

Here $\delta(t)$ denotes the Dirac distribution. Using the properties of $\delta(t)$ one can write the right side of (1) as

$$(s_l \tau, p) = \int_{-\infty}^{+\infty} s(x, \tau + px) dx \quad (2)$$

Then the direct slant-stack defines a correspondence between the (x, t) and (τ, p) domains. Hence the image of the line $t = \tau + px$ is the point $M(\tau, p)$ in the τ - p plane. Notice that the linear of a refraction line $t = \tau + px$ is represented by the point $M(1/V, t_0)$ and the slant stack transforms the reflection hyperbole into an ellipsis.

5 - CONVENTIONAL FILTERING

The application of this method on an organised event will result in another organised event in the (τ, p) domain. Thus, for example, a parabola in the (x, t) domain becomes an ellipsis in the (τ, p) domain. The application of this transformation for multiples suppression is interesting because it decomposes a seismic event according to its apparent velocity. In this case, a long multiple will take a more stretched form due to its low velocity. The suppression of multiples in this domain requires particular attention and precision for the definition of the undesirable zone. Such zone is generally based on velocity analyses, while the filtering is performed by a simple mute. It is not evident to suppress totally the energy of the multiples especially when it is superposed to the energy of the simple reflection in the (τ, p) domain; this will induce the elimination of the primary energy by multiple suppression.

5-1 - The non linear filter

To measure the degree of event separation in the linear (τ, p) domain, we applied the non linear filter. We use solely the predicted multiple model (M. Benchikh and M. Djeddi, 2002; R. Sellam and al., 2005).

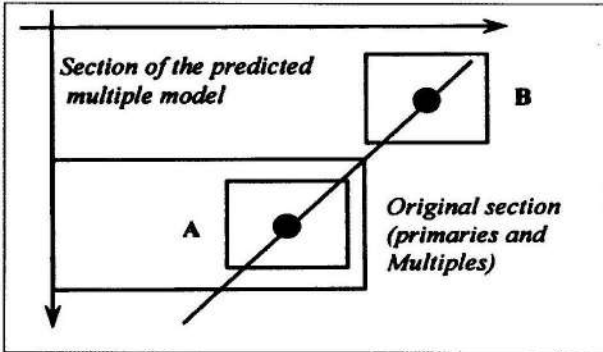


fig. 3 - The design of the non-linear filter
Principe du filtrage non linéaire

The non linear filter is used as an energy comparator between the original model and the multiple model. This filter is non linear as its function is dependent on the input data A.

5-2 - Butterworth function of gain

Selecting the function of gain of the non linear filter is arbitrary. The Butterworth function is used in the present work; its formula is the following:

$$G(\tau, p) = \frac{1}{\sqrt{1 + \frac{B(\tau, p)^n}{\varepsilon \cdot A(\tau, p)^n}}} \quad (3)$$

where $B(\hat{\omega}, p)$ is the sum on a two dimensional window of the amplitude spectrum of the centred sample of the predicted multiple model, and $A(\hat{\omega}, p)$ the amplitudes obtained after $\hat{\omega}$ -p transformation of the original record spectrum; n is the parameter used for the smoothing control of the filter, and ε the elimination parameter of the multiple that is related to reflection coefficients.

The value of n controls the smoothing of the amplitude spectrum so that to avoid the Gibbs phenomenon (fig. 4). However this will result in a poor filtering. Using a great value of n will improve the filtering but will cause stiff ramps.

It will be necessary therefore to determine carefully this parameter to avoid such disadvantage.

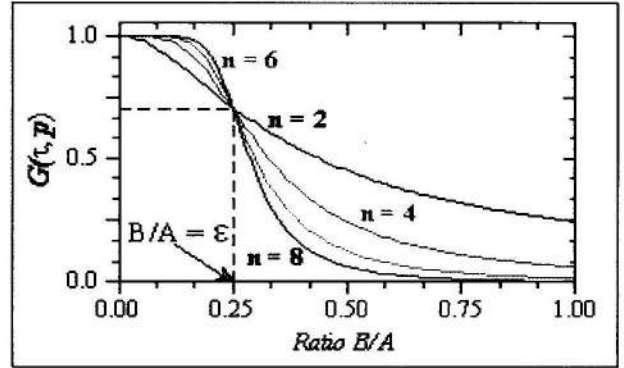


fig. 4 - Gain function of the non linear filter
Fonction de gain du filtre non linéaire

6 - PARABOLIC TRANSFORM

On a CMP or CSP film, events tend to be rather hyperbolic than linear. The linear transform is not very efficient for the suppression of the organised noise (multiple), or the separation of P and S waves which have also hyperbolic «forms» in the (x, t) domain. Contrary to the linear $\hat{\omega}$ -p transform, the parabolic $\hat{\omega}$ -p transform can be applied to transform hyperbolic events of a seismic section into points. This parabolic $\hat{\omega}$ -p transform consists in the summation of samples according to a parable of equation $t = \tau + px^2$ resulting in a much better concentration of energy hence a much better separation of events (Arthur, B. and Weiglein, 1999).

6-1 - Continue parabolic transform

The following is a derivation of the parabolic $\hat{\omega}$ -p transform definition. To obtain a fair resolution image with the parabolic $\hat{\omega}$ -p transform, we proceed as in the linear τ -p transform case (version 2).

The inverse parabolic τ -p transform is defined as:

$$u_p(x,t) = \int_{-\infty}^{+\infty} v_p(p,t - px^2) dp \tag{4}$$

where $\tau = t - px^2$ $v_p(p,t - px^2)$ and denotes the direct image $\hat{\delta} - p$ transform.

In the frequency domain this transforms is written as.

$$U_p(x,\omega) = \int_{-\infty}^{+\infty} u_p(x,t) e^{-i\omega t} dt \tag{5}$$

According to the expression (4) we obtain.

$$U_p(x,\omega) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} v_p(p,t - px^2) e^{-j\omega t} dt \tag{6}$$

Expression (6) can take the following form.

$$U_p(x,\omega) = \int_{-\infty}^{+\infty} V_p(p,\omega) e^{i\omega p x^2} dp \tag{7}$$

We define a direct projection function labelled $V_p'(p,\omega)$ as by putting.

$$V_p'(p,\omega) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} V_p(p',\omega) e^{i\omega(p-p')x^2} dp' dx \tag{8}$$

From equation (8), it can be define a new function $\rho(p,\omega)$ under the form.

$$\rho(p,\omega) = \int_{-\infty}^{+\infty} e^{i\omega p x^2} dx \tag{9}$$

Replacing the equation (9) in (8) we get:

$$V_p'(p,\omega) = \int_{-\infty}^{+\infty} V_p(p',\omega) \rho(p - p',\omega) dp' \tag{10}$$

One can recognize throw expression (jus) the convolutive form.

$$V_p'(p,\omega) = V_p(p,\omega) \underset{\circ}{\ast} \rho(p,\omega) \tag{11}$$

where $\underset{\circ}{\ast}$ denotes the convolutive product relatively to the frequency variable ω .

The equation (11) represents the standard normal equation of convolution according to the p variable. It can be solved using the Levinson fast algorithm. To obtain a direct parabolic $\tau - p$ transform corresponding to the inverse of the transformed (equation 2), a directional deconvolution following p is to be applied. There are two cases which can be considered for the calculation of the convolution operator (9) :

- The interval of integration is infinite

$$x \in]-\infty, +\infty[$$

- The interval of integration is finite

$$x \in [x_{\min}, x_{\max}]$$

From the tests done on the operator of directional convolution, we can conclude that this operator gives a fairly better concentration of energy in the ($\hat{\delta} - p$) domain resulting in a more effective separation of events in this domain. It also allows us to recover the frequency content of the signal during the inverse transform.

6-2 - Conventional filtering using the parabolic $\hat{\delta} - p$ transformation

The parabolic $\hat{\delta} - p$ transform leads to either better separation or energy concentration in this domain. A hyperbolic event approximated by a parabola in the (x,t) domain will appear as a point in the (τ, p) domain. In this case, the parameter is no more the p ray parameter, but the value $1/(2V_0^2 t_0)$, V_0 being the velocity of the area and the double time in normal incidence.

In this new domain the multiple will be represented by a point on values of p greater than that of simple reflections. This is related to the fact that the velocity of the multiple is lower than that of the primary reflections. Such a particular feature will allow us to distinguish reflections from multiples, and consequently facilitate multiple suppression.

With this filter, multiples are attenuated by a mute law of the undesirable zone. This law is obtained from primary reflection velocities and their position in double time. The effective good separation of events offered by this new method, makes multiple suppression easier than in the (x, t) , (f, k) and linear (τ, p) domains.

6-3 - Non linear parabolic τ - p filtering

In the parabolic (τ - p) domain, the multiple attenuation can be achieved merely via the elimination of its energy. The used filter is based on the predicted multiple model. The advantage when applying this two dimensional filter in the parabolic (τ - p) domain is the automatic determination of the portion to eliminate (the zone occupied by multiples), which is done by comparison of energies between the predicted multiple film and the input original model. The gain function applied previously in the (f, k) domain is used once more for the non linear filtering in the parabolic (τ, p) domain.

Problems of approximation

Additionally to the problems due to the data discretisation as well as to the finite dimension of our pattern, there is another problem relating to the use of the parabolic (τ, p) transform: the approximation of a hyperbole by a parable.

Contrary to the slant stack (linear τ - p transform) which uses filters to reduce the effect of truncation (End Effect), in the parabolic addition, mathematical techniques are applied to solve the problem. Such techniques consist in transforming a hyperbolic event into a parabolic one. Among these methods, we can quote the t^2 -stretching method developed by Yilmaz (Yilmaz Özdoğan, 1987) and the residual NMO (RNMO) correction method (Hampson-1986). Further to these transformations, the p parameter will have no longer the same physical meaning.

For a sound understanding of the effect caused by the approximation of a parable to a hyperbole as well as the usefulness of RNMO in the parabolic (τ, p) domain, a synthetic example is provided on figure 1 which illustrates a parabolic transformation of four hyperbolas of reflections situated at different times.

7- RESULTS AND DISCUSSION

7-1- Models used

To perform $f - k$ and linear $\tau - p$ filtering, we used a model with horizontal layers (model no1), then we modified it by introducing a dip in the first layer (model no2) to evaluate the efficiency of these filters.

The third model (fig. 6) is chosen especially for the parabolic τ - p filter. The multiples are completely superposed to primary reflections even for large offsets.

Hereafter are given the thickness and velocity of each layer for the three models:

| Model N°1 | | Model N°3 | |
|---------------|----------------|---------------|----------------|
| Thickness (m) | Velocity (m/s) | Thickness (m) | Velocity (m/s) |
| 225 | 1500 | 550 | 2500 |
| 300 | 2000 | 660 | 3000 |
| 250 | 2500 | 770 | 3500 |
| 320 | 3800 | 880 | 4000 |
| 380 | 3200 | | |

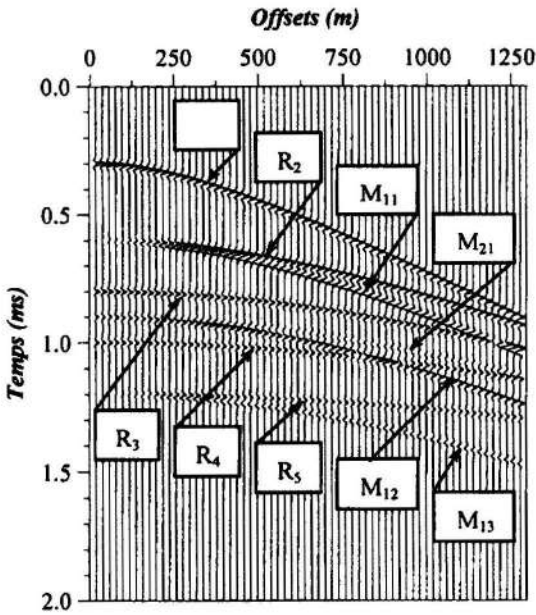
The bedrock velocity is equal to 4500m/s for all models.

Four multiples have been generated; three for the first layer and one for the second one.

The second model shares similar characteristics with the first one. Its first layer exhibits a dip at low angles.

Three multiples of first, second and third order are generated for the third model.

**7-2 - Linear $\tau - p$ filter
interpretation of results**



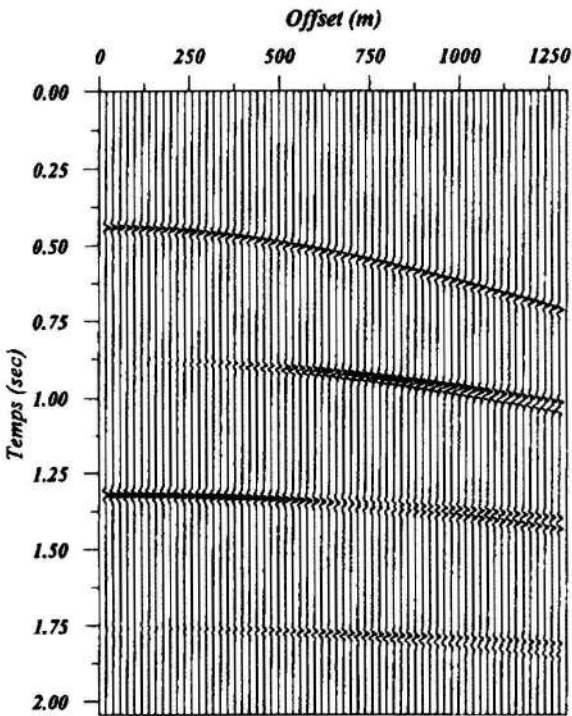
**Fig. 5 - Model with horizontal layers
Modèle à couches horizontales**

Figures 8 through 11 present the linear τ - p filtering results for the models 1 and 2. As in the case of the $f - k$ filtering, we used both the conventional and the non linear filter with the model contaminated with noise.

In the two cases (horizontal and inclined layers), this conventional method leads to a lower multiple attenuation than the $f - k$ filtering. This is due to the poor separation between the multiple and the simple reflection in the (τ - p) domain. We get similar results even when the events are well separated in the (x, t) domain (figures 8a and 12a).

Recurring to the use of the non linear filter in the (τ, p) domain proved to be inefficient in this case. Indeed, when it suppresses multiple energy it does also eliminate part of the simple reflection energy.

**7-3 - Parabolic filter
interpretation of results**



**Fig.6 - Third model
Troisième modèle**

In such delicate case, multiples superpose completely to primary reflections till reaching the large offsets. The model has been especially selected to measure the efficiency of the automatic definition of the undesirable zone via parabolic filtering. Obtained results are illustrated in figures 6 and 7.

Satisfactory results are obtained via the conventional filtering using a simple mute in the parabolic ($\tau - p$) domain. It is interesting to highlight recovery of the primary reflection energy which was superposed with the multiple energy.

The use of the non linear filter leads to similar satisfactory results.

8 - CONCLUSION

The non linear parabolic τ -p transform is a good technique for multiple suppression. It is more efficient than both the f-k conventional filter and the linear τ -p transform. Indeed:

- This method ensures much better separation between multiples with primary reflections in the filtering domain.
- It is efficient for both large and short offsets.
- It offers also the possibility to recover the primary reflection energy.

This technique is particularly efficient for deep oil objective; in this case the approximation of the hyperbole in a parable is highly satisfactory.

The non linear filtering using the predicted multiple model is also a recent method which is increasingly gaining importance in seismic, namely in the field of multiple suppression.

This filtering will be relatively better either when the multiple energy is superposed to that of the primary reflection, or in the presence of the aliasing phenomenon.

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-ANNEXE-

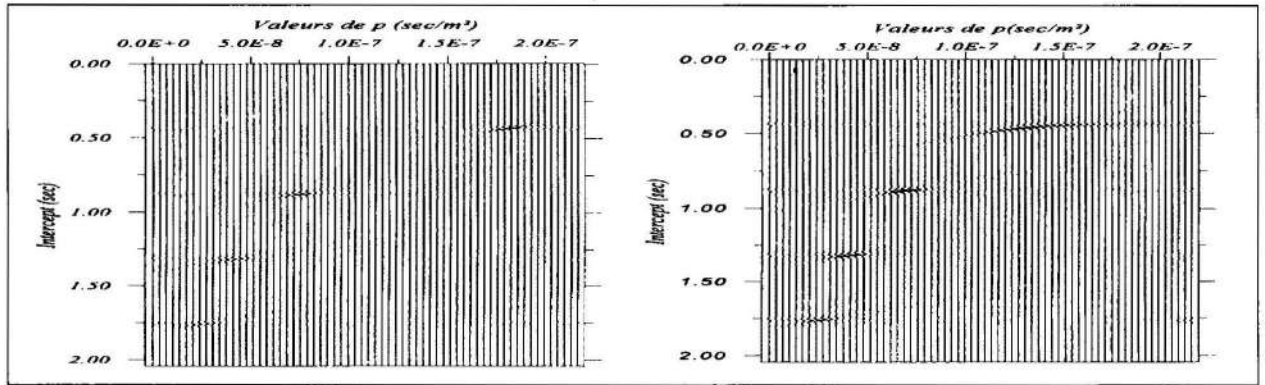


Fig. 7 - Usefulness of RNMO in filtering using parabolic $\tau - p$ transform
Importance du RNMO dans le filtrage par la transformation $\tau - p$ parabolique

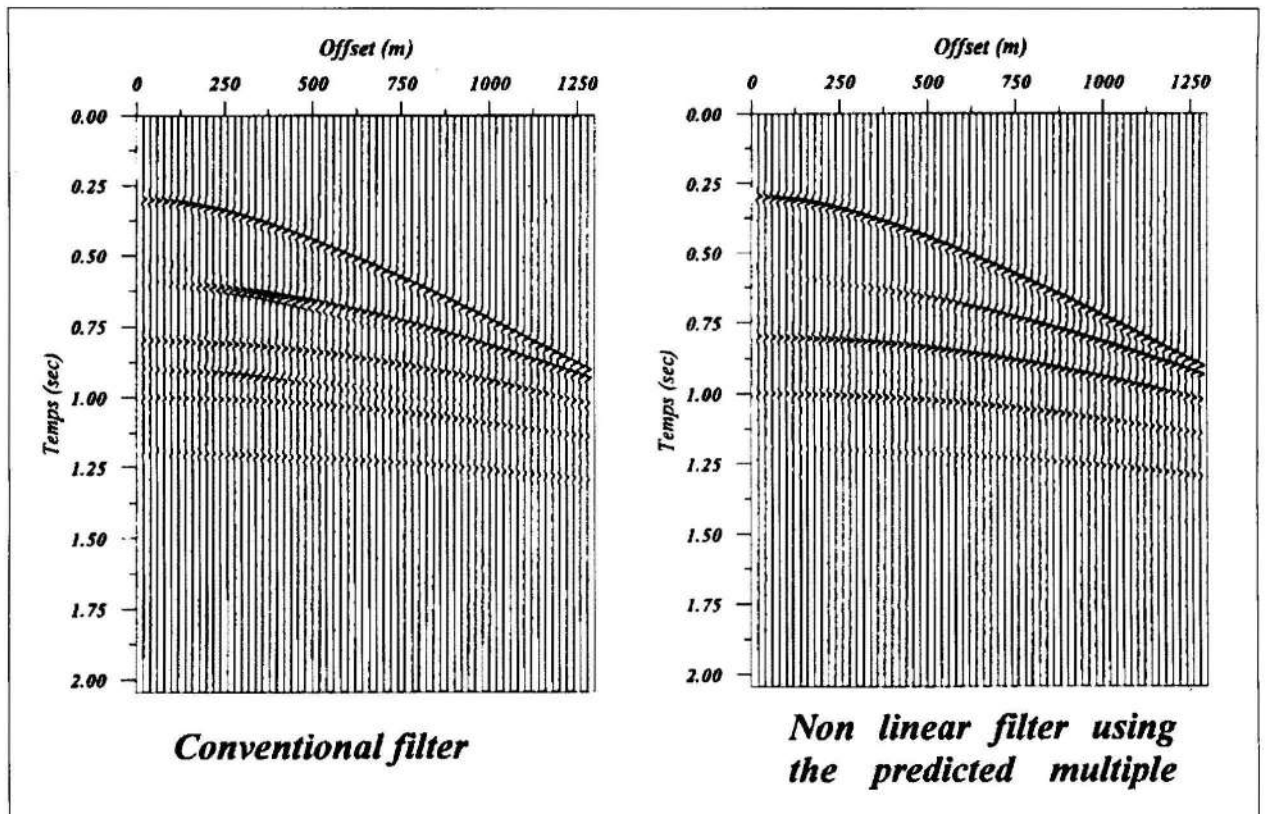


Fig. 8 - Linear $\tau - p$ filtering using model n°1
Filtrage par la $\tau - p$ linéaire (model n°1)

MULTIPLES SUPPRESSION USING NON LINEAR FILTERING IN (τ , p) DOMAIN

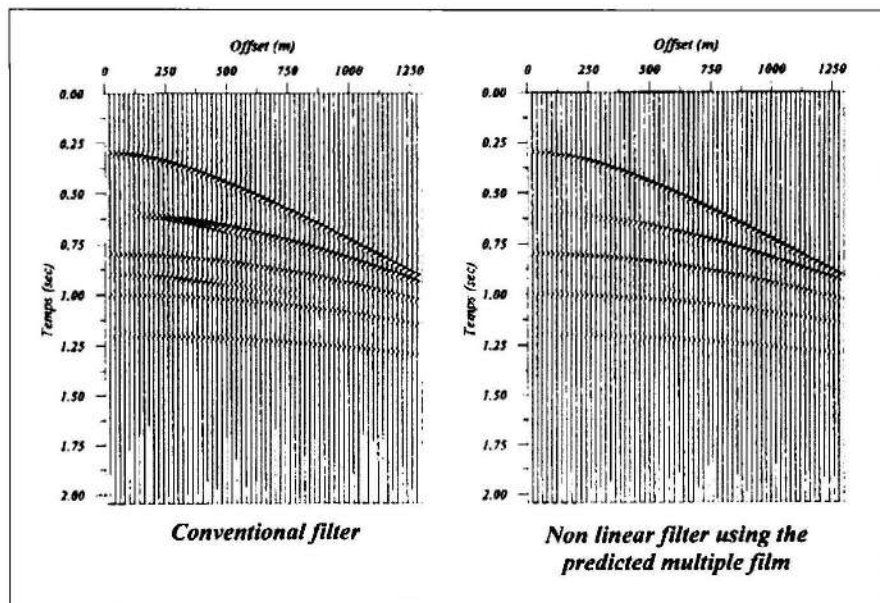


Fig. 9 - Linear τ - p filtering using model n°1 with 20% of noise

Filtrage τ - plinéaire appliqué sur le modèle n°1 avec 20% de bruit blanc

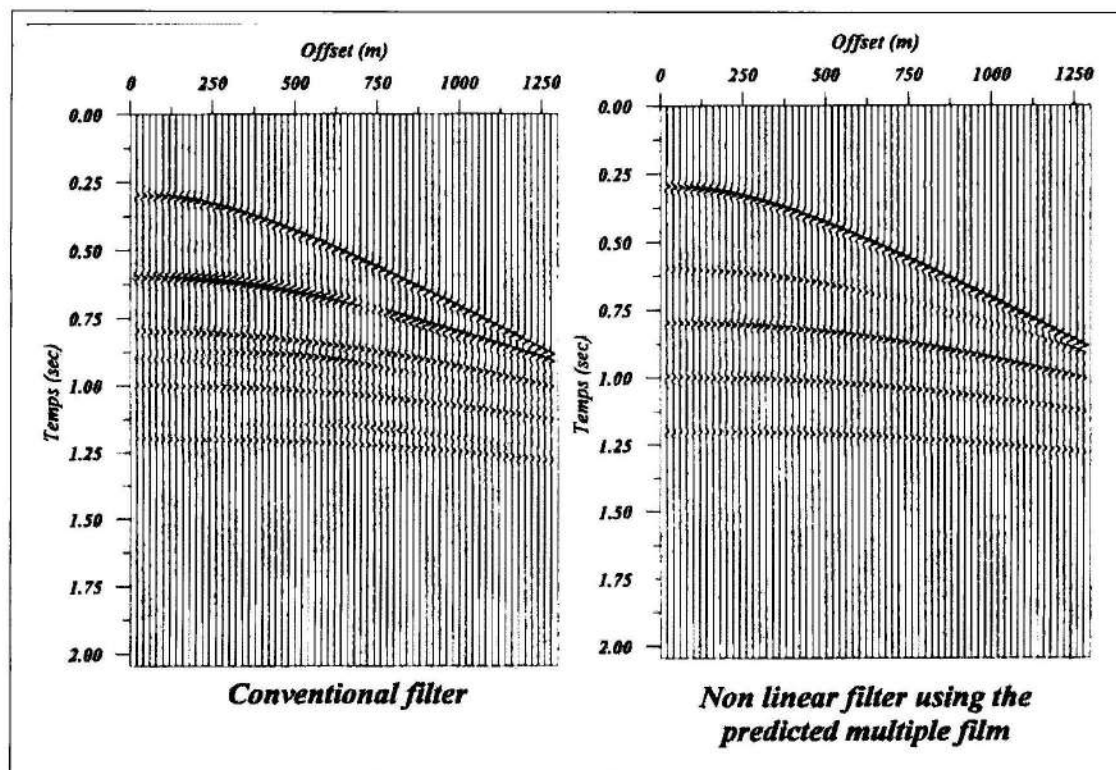


Fig. 10 - Linear τ - p filtering using model n°2

Filtrage τ - plinéaire appliqué sur le modèle n°2

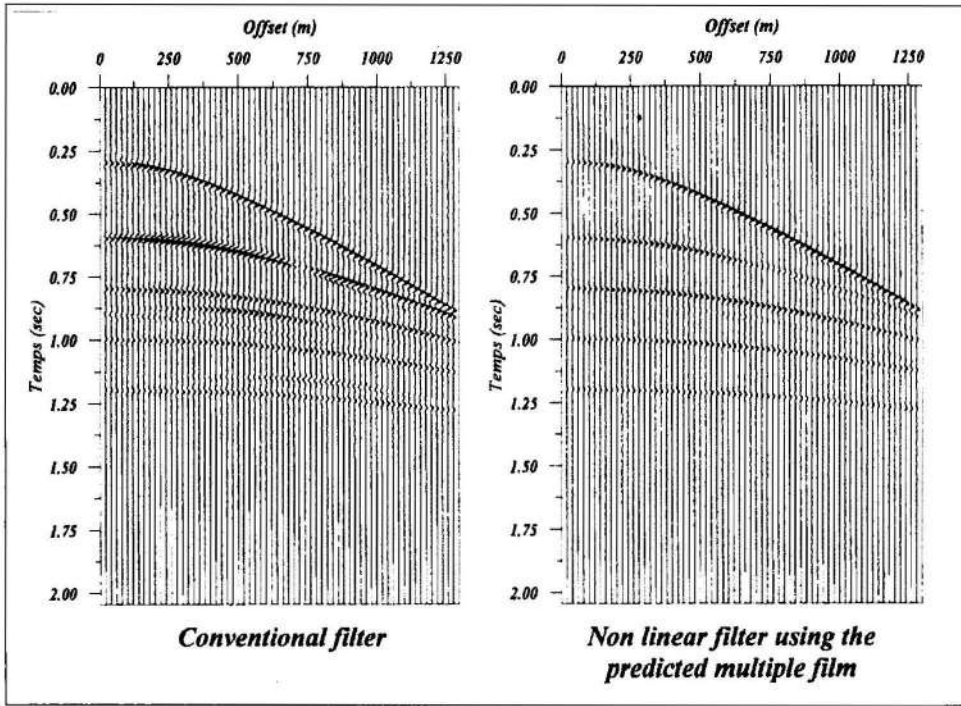


Fig. 11 - Linear $\tau - p$ filtering using model n°2 with 20% of white noise
 Filtrage $\tau - p$ linéaire appliqué sur le modèle n°2 avec 20% de bruit blanc

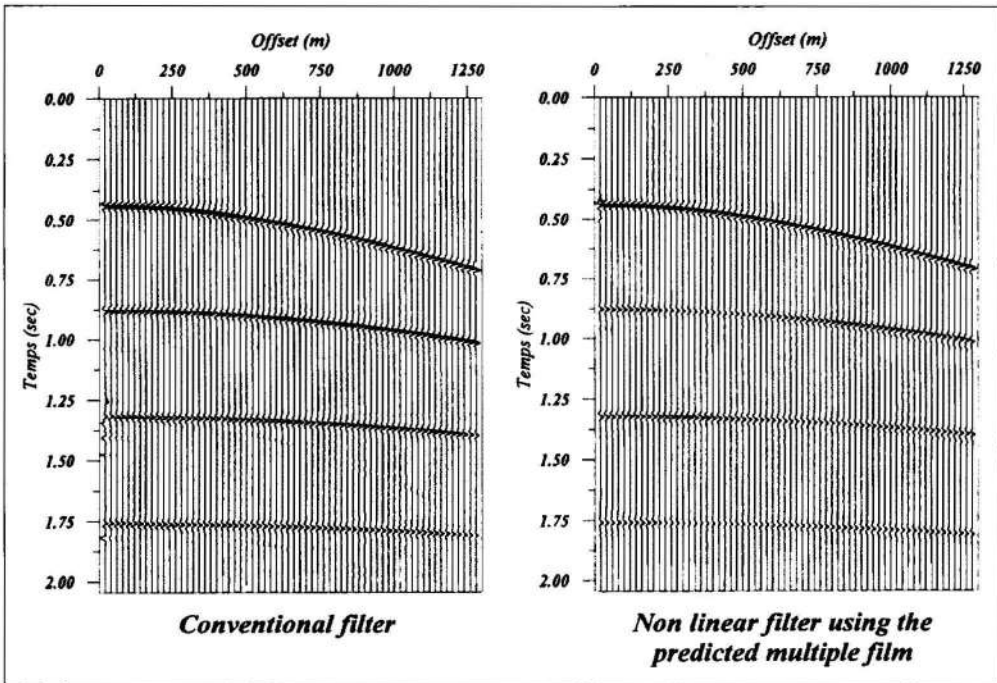


Fig. 12 - parabolic $\tau - p$ filtering using model n°3
 Filtrage $\tau - p$ parabolique appliqué sur le modèle n°3

MULTIPLES SUPPRESSION USING NON LINEAR FILTERING IN (τ , p) DOMAIN

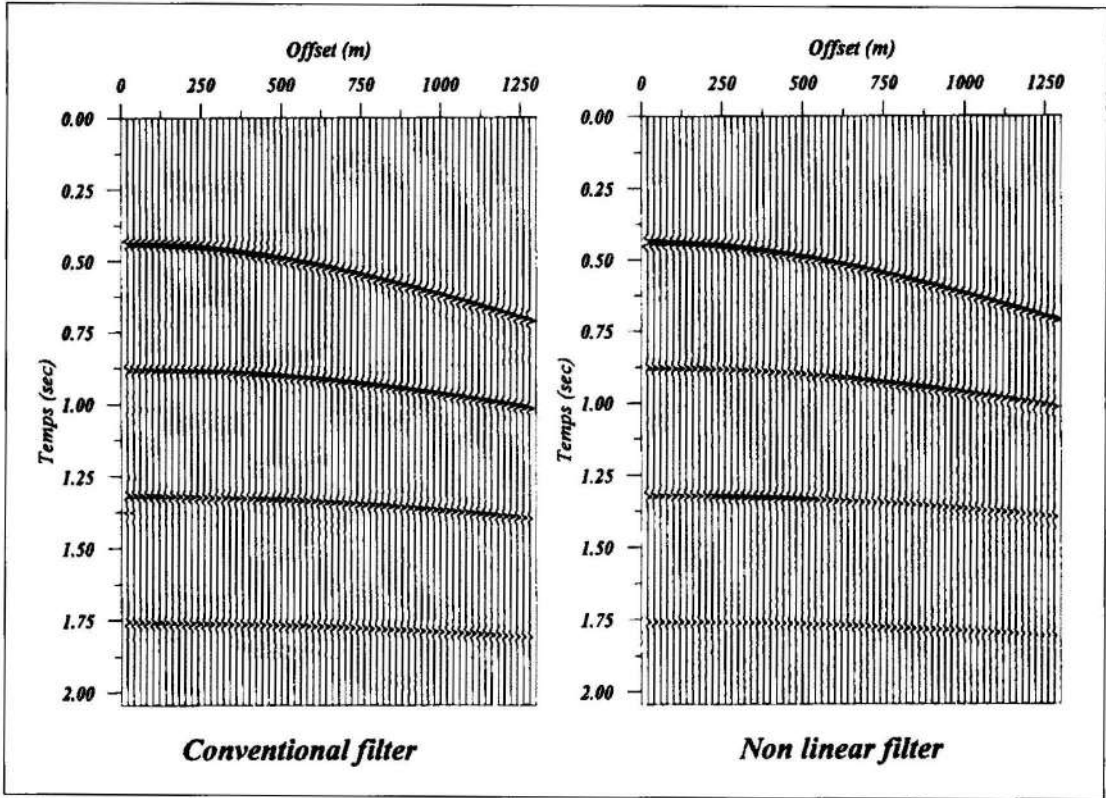


Fig. 13 - Parabolic τ - p transform filtering using model n°3 with 20% of white noise

Filtrage parabolique utilisant le modèle n°3 ave 20% de bruit blanc