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QUANTITATIVE INTERPRETATION OF VLF-RESISTIVITY DATA TRANSFORMATION.

Mohamed DJEDDi*, Hayder Aziz BAKER and Hakim ZAKOUR***

ABSTRACT

A new method for interpreting quantitatively the VLF-Resistivity data is introduced. It depends on the transformation of VLF-R measurements into VLF-EM ones. The calculated result is called VLF-C. The transformation is based first on the Maxwell equations. A linear filtering technique is then applied to the VLF-C data in order to obtain an estimation of length, width and depth of the causative structures. Tests on field data over a buried wall gave dimensions estimation which is in a very good agreement with the actual ones.

Keywords - VLF-R - MT-VLF - Electromagnetic resistivity - Archaeological prospecting- Near-surface geophysics.

INTERPRÉTATION QUANTITATIVE DE LA TRANSFORMÉE VLF - RÉSISTIVITÉ.

RÉSUMÉ

Une nouvelle méthode d’interprétation quantitative des données VLF-R (MT-VLF) est présentée. Elle est basée sur la transformation des mesures de VLF-R en mesures de VLF-EM. Le résultat ainsi obtenu est appelé VLF-C. La transformation est essentiellement basée sur les équations de Maxwell. Une technique de filtrage linéaire est alors appliquée aux données de VLF-C afin d'estimer les dimensions des structures causatives. Les essais sur des données de terrain obtenues sur un mur enterré ont permis de déterminer les dimensions de la structure qui sont en très bonne concordance avec les dimensions réelles.

Mots clés - VLF-R - MT-VLF - Résistivité électromagnétique - Prospection archéologique- Géophysique de surface.

*Département de Géophysique, FSTGAT, USTHB, BP. 32, Bab Ezzouar Al-Alia, Alger, Algeria.
**Department of Geology, Faculty of Science, UAE University, P.O. Box 17551, Al-Ain, UAE.
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INTRODUCTION

The VLF method is particularly one of the most rapid and low cost mapping techniques. It is widely used in mineral exploration (Paterson and Ronka, 1971; Phillips and Richards, 1975). However, its application in shallow engineering, environmental study and archaeological site investigations has received less attention (Linford, 2006; Drahor, 2006; Djeddi and *al*., 1999; Djeddi and Baker, 2008, and 2010; Al-Tarazi and *al*., 2008). The VLF-EM method uses the transmitted signals of radio communication stations as a primary field (10-30 kHz). The primary source can be considered as an electric dipole which has three spherical components. At a very great distance from the emitter, the plane-wave approximation holds true and the electromagnetic field may be considered as uniform. It is then characterized by H_y, the horizontal component of the magnetic field and E_x (resp. E_z) the horizontal (resp. vertical) component of the electric field (fig. 1).

It is known that whenever a subsurface discontinuity is encountered, the intensity of the Ex varies. When this discontinuity is a conductor, the primary field (H_p) induces an electric current and a secondary magnetic field (H_s) will be generated. Therefore, two very well established techniques are generally used in fieldwork. The first is the VLF-EM in which the real and imaginary parts of the ratio H_z/H_y are measured, where Hz is the vertical component of H_s, and H_y is the sum of the primary magnetic field and the horizontal component of H_s. The second is the VLF-R, in which the orthogonal components of the horizontal electric and magnetic fields are measured. Their quotient can easily be transformed into apparent resistivity using the classical definition given by Cagniard (1953).

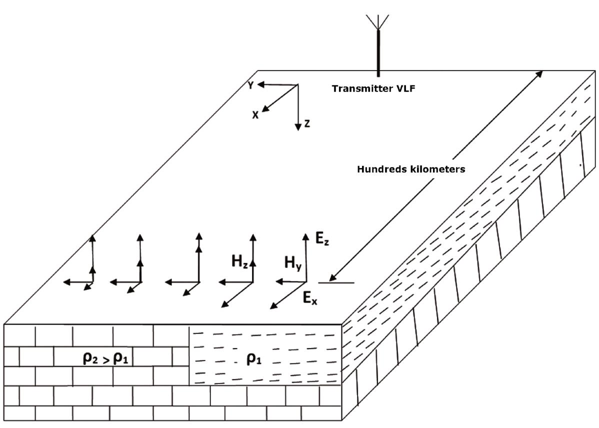

Fig. 1 - The electromagnetic wave on heterogeneous medium.

Onde électromagnétique au dessus d'un milieu hétérogène.

$$\rho = 1/\mu\omega\ (E_x/H_y)2$$

Where μ is the magnetic susceptibility, ω is the angular frequency of H_p, E_x is the electrical field in the X-direction and H_y is the magnetic field in the Y-direction.

The response normally obtained with VLF-EM surveys over conducting mineralised zones is a sine-curve anomaly. The real and imaginary parts of the resultant field, commonly plotted on profiles, can only be interpreted to give the position and the possible depth of the target. However, the problem of delineating the subsurface structures is also of great interest in mining geophysics. Fraser (1969) proposed a filter in which the cross over point of a VLF-EM profile is transformed into a maximum. Then all the transformed profiles are presented on a map, and maximum value features are delineated, showing the horizontal location of conducting targets, but it does not give the dimensions. Djeddi et *al*. (1998) using a linear filtering technique, gave the possibility of delineating any 3D target, and also showed that the map resulting from Fraser's filter can be used to calculate the dimensions of the subsurface structures.

The VLF-EM technique has generally been used to locate conducting targets and the sensitivity of this technique to small resistivity changes is very weak. Therefore, the VLF-R is more convenient for mapping lateral resistivity changes. But at present, the interpretation of the VLF-R data is still limited to two-layer interpretation and in case of horizontal bedding, or simply to locate the causative structures. Moreover, the dimensions of these structures are not determined (Sharma and Kaikkonen, 1998; Kaikkonen and Sharma, 1998).

In this paper, we propose an approach of interpreting quantitatively the VLF-R profiles over any structure, regardless of their resistivity values. The first step is to transform the VLF-R measurements into VLF-EM data. The transformation filter is based on the Maxwell's equations. Then, by applying a linear filtering technique, the dimensions and depth of the target can be obtained.

In addition, the proposed procedure in this work can be extended to interpret VLF-Resistivity maps of any 3D structure.

Few papers on the transformation of VLF-EM data into apparent resistivity have been published (Gharibi and Pedersen, 1999; Chouteau and *al*., 1996). The works are, in general, based on transforming the measurements of VLF-EM profiles into VLF-R ones for 2D structures, in order to compare the obtained results with other electromagnetic and electrical resistivity data. However, no quantitative interpretation approach was given or suggested. Moreover, Chouteau and *al*. (1996), as a conclusion, recommended the use of the VLF-R method, since VLF-EM method is not very sensitive to small resistivity changes in a resistive medium.

In our approach and regardless of the resistivity values, quantitative results can always be obtained from the transformed VLF-EM data. This makes the new approach of great importance especially in the field of archaeological and civil engineering.

THEORY OF TRANSFORMATION

Considering the TE mode, in which the horizontal magnetic field (H) is oriented along the Y-axis and the electrical field along the X-axis; and that the target has a strike following the X-direction. In a Cartesian reference system, where Z is pointing downward, the general relationship between the electric field and the magnetic field is given by Maxwell's equations.

$$\nabla\times E = -\delta B/\delta\tau \quad (1)$$

where **E** is the electric vector and **B** the magnetic induction vector.

Assuming that there are no changes in electrical resistivity of the subsurface along the X-axis, then equation (1) can be written as follows

$$\delta E_x/\delta_y = \delta B_z/\delta t \quad (2)$$

Assuming that the time dependence of the magnetic field is of the form

$$B_z = \mu\ H_o \exp(\ i\omega t) \quad (3)$$

equation (2) becomes

$$\delta E_x/\delta_y = i\omega\mu H_z \quad (4)$$

where ω is the angular frequency and μ the magnetic permeability.

Now, assuming that the primary magnetic field (H_y) does not change significantly along the Y-axis, therefore, it can be considered as constant, we obtain

$$i\omega\mu H_z/H_y = \delta(E_x/H_y)/\delta y \quad (5)$$

Using the Cagniard (1953) approach, and assuming that each measurement senses a homogeneous ground directly below it (Chouteau and *al.*, 1996), we can write equation (5) for a measurement at the earth's surface in this way:

$$i\omega\mu H_z/H_y = (i\omega\mu)^{1/2} \; \delta(\rho^{1/2})/\delta y \quad (6)$$

or

$$\delta(\rho^{1/2})/\delta y = (i\omega\mu)^{1/2} H_z/H_y \quad (7)$$

If we consider the electric dipole theory in VLF-EM method, the positive vertical field is pointing upwards at the earth's surface. Therefore, the sign of the right-hand side of equation (7) must be changed in order to represent this orientation. This gives

$$\delta(\rho^{1/2})/\delta y = -(i\omega\mu)^{1/2} H_z/H_y \quad (8)$$

H_z/H_y is a complex number and can be written as

$$H_z/H_y = P + iQ \quad (9)$$

where P=Real (H_z/H_y) and Q=Imag (H_z/H_y).

The right-hand term of (7) becomes

$$-(i\omega\mu)^{1/2} H_z/H_y = -(\omega\mu/2)^{1/2} [(P-Q) + i(P+Q)] \quad (10)$$

In theoretical modelling and in practical tests, the imaginary part Q which represents the eccentricity of the polarisation ellipse (Paterson and Ronka, 1971) is less reliable and it is usually smaller than P and therefore, it can be neglected. In this case we can write equation (10) as

$$-(i\omega\mu)^{1/2} Hz/Hy = -(\omega\mu/2)^{1/2} (1+i)P \quad (11)$$

On the left-hand side of equation (7), ρ is real, and in order to force the right-hand term to be also real and in using equations (8), equation (7) becomes.

$$-1/(\omega\mu)^{1/2} \, . \, \delta(\rho^{1/2})/\delta y = P \quad (12)$$

Equation (12) shows that it is possible to obtain the VLF-EM data (P) by the differentiation of the square-root of the resistivity profile with no imposed restrictions concerning the value of the resistivity. This makes the present approach practical and easy to use in all situations.

Furthermore, this approach may help in giving a detailed quantitative interpretation of 3D structures located by the VLF-R method. It can easily be shown that in the differentiation of any initial function, each maximum is transformed into zero value and each inflexion point into peak or trough (Djeddi et *al.*, 1998). Therefore, the calculated VLF-EM map can be transformed in the X- and Y-directions and used in such a way that the obtained peaks and troughs give the exact dimensions of the body. In such case, equation (12) becomes:

$$\delta P/\delta x = -1/(\omega\mu)^{1/2} \, . \, \delta^2(\rho^{1/2})/\delta x\delta y \quad (13)$$

$$\delta P/\delta y = -1/(\omega\mu)^{1/2} \, . \, \delta^2(\rho^{1/2})/\delta y^2 \quad (14)$$

APPLICATION TESTS

1- Synthetic data

To obtain a synthetic 2D data set, we have first considered a conducting thin elongated structures (2x20x2 metres) (ρ_2) embedded in a resistive homogeneous medium (ρ_1) overlying a very resistive substratum (ρ_3). The second model is a resistive body with a same dimension embedded in a conductive medium. The substratum in the second case is identical to the first one (fig. 2). The apparent resistivity is computed at 20 kHz using a modified integral equation modelling program, given by Tabbagh (1989). In this program, the electromagnetic integral equations are approximated by dividing the body into number of cells, where the field in each cell is assumed constant and equal to its value at the centre of the cell. The value of the electromagnetic field at the surface of the earth is computed by adding contributions from all cells. Figures 3a and 4a show the apparent resistivity profiles obtained over conducting and resistive bodies respectively, both having extension along the direction of Ex.

By applying the derivative filters to the data presented in figures 3a and 4a, the resistivity profiles will be transformed into VLF-EM real component ones (dashed line) named here as

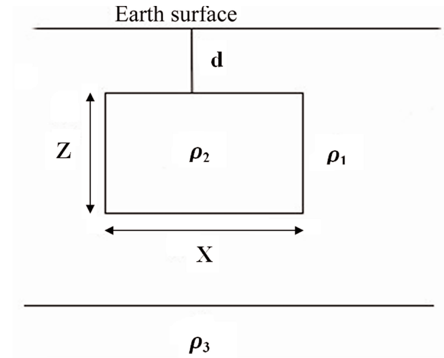

Fig. 2 -Schematic presentation of the models used in calculating the electromagnetic resistivity anomalies.

Représentation schématique du modèle utilisé pour le calcul de l'anomalie de résistivité électromagnétique.

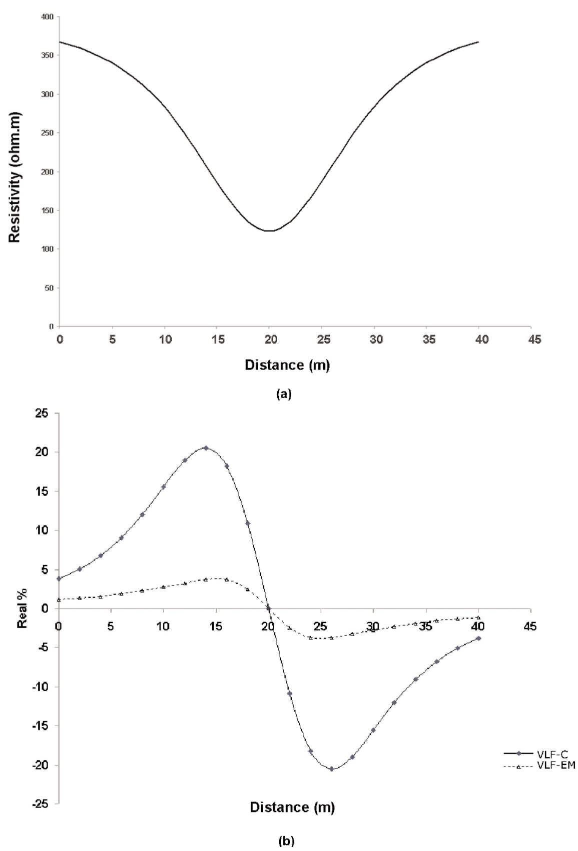

Fig. 3 - a) Electromagnetic-resistivity profile over a conducting structure ***(Profil de résistivité électromagnétique au dessus d'une structure conductrice)***.

b) VLF-C and VLF-EM anomaly over the conducting structure ***(Anomalie VLF-C et VLF-EM au dessus de la structure conductrice)***.

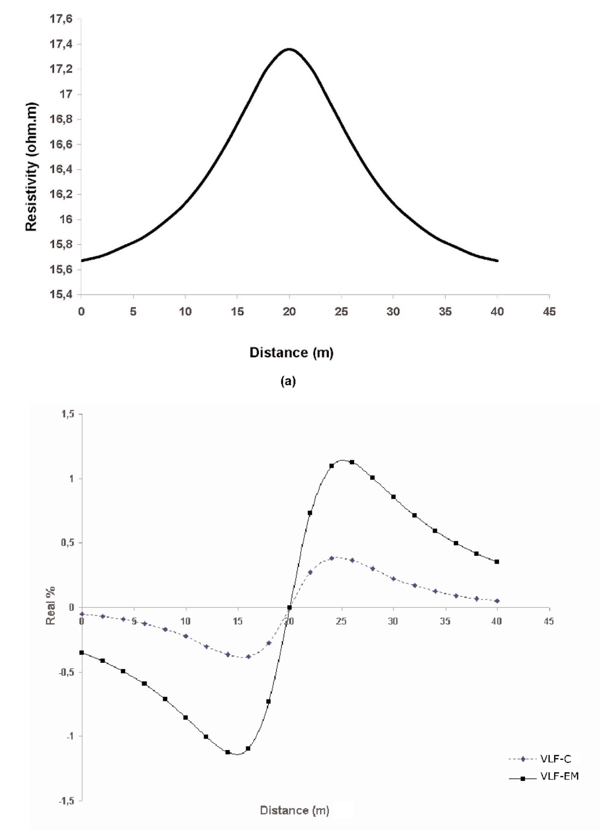

Fig. 4 - a) electromagnetic-resistivity profile over a resistive structure ***(Profil de résistivité électromagnétique au dessus d'une structure résistante)***.

b) VLF-C and VLF-EM anomaly over the resistant structure ***(Anomalie VLF-C et VLF-EM au dessus de la structure résistante)***.

VLF-Calculated (VLF-C). The transformed profiles show the peaks and troughs that are commonly observed with VLF-EM anomalies.

A comparison is being made between the VLF-C real component and the VLF-EM real component that has been calculated over the same conducting structure. It can be clearly noticed that there is a very good coincidence between the two real component curves. However, the observed slight deviation (15% for a conductor body and 1% for a resistive) from a perfect matching between the two curves is an expected one and it is mostly due to the application of the filter, which was built after some approximations.

The most important for the VLFC is not the amplitude of the anomaly but the difference in distance between the extrema and their ratio and also the position of the inflection point, then the gap between the curves measured and calculated may be neglected.

The VLF-C anomaly can now be used to estimate the dimensions and the depth of the subsurface structures; i.e. quantitatively interpreting the VLF-R anomaly. This can easily be done using one of the published approaches (Paterson and Ronka, 1971; Djeddi and *al*., 1998).

2- VLF-R maps

Since there is no direct quantitative interpretation of VLF-R maps, the proposed transformation technique has been tested on VLF-R maps in order to derive the dimensions of the causative structures thanks to Djeddi et *al*. (1998) approach. The location of the anomelous structure is shown in all subsequent figures 5 to 8.

To obtain the dimensions and depths of any causative structures, the VLF-R map is first transformed into VLF-C map (figs. 5 and 6). Then, by simply applying the derivatives in both the

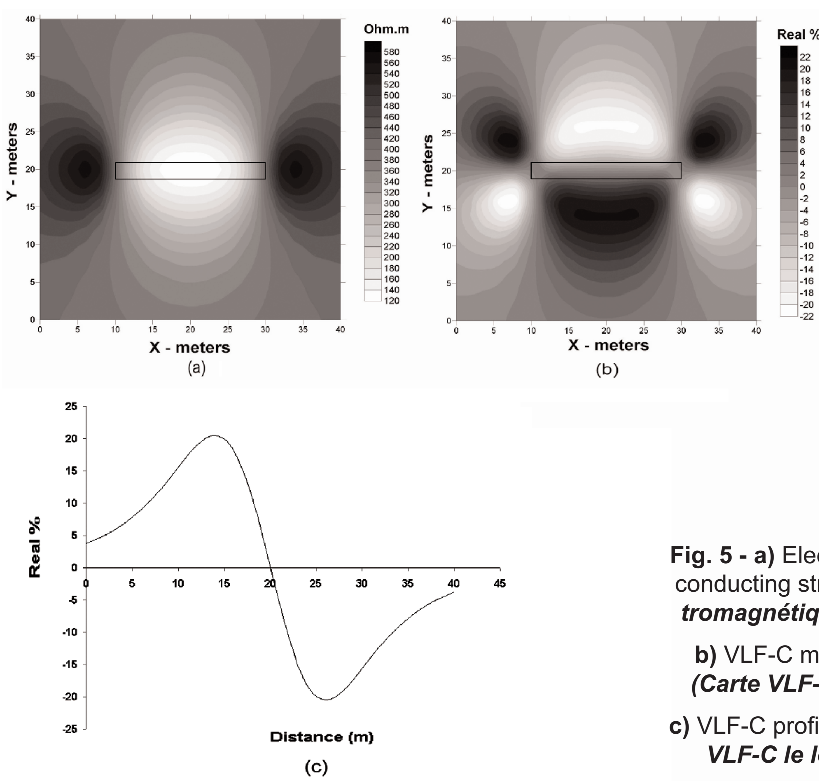

Fig. 5 - a) Electromagnetic-resistivity map over a conducting structure ***(Carte de résistivité électromagnétique de la structure conductrice)***.

b) VLF-C map over the conducting structure ***(Carte VLF-C de la structure conductrice)***.

c) VLF-C profile along Y-direction at X=20 ***(Profil VLF-C le long de la direction Y à X=20)***)

X- and Y-directions, the dimensions of the subsurface structure can be calculated. In the case of the X-derivative, the VLF-C map (having peak and trough) will be transformed and give four extrema (two peaks and two troughs) (fig. 7 a1 and b1) that indicate the approximate limits of the causative body in the XY plane. The distance between the peak and trough along the X-direction gives the exact dimensions of the body along this axis. The distance between the two extrema along the Y-direction, which is in fact the same as that of the peak-to-peak anomaly observed normally with a typical VLF-EM anomaly, can be used to estimate the depth to top of the structure.

The extrema along the Y-derivative of the VLF-C map can be used to determine the limits of the body in the Y-direction (fig. 8 a1 and b1). To obtain the exact dimension, a profile has to be drawn passing through the extrema, and in a similar way to that used in the gravity method (Bott and Smith, 1958), the half-value of the central intensity would give the exact extension of the body in the Y-direction (fig. 8 a3 and b3).

It is important to indicate here, that the X-direction for all the derivatives must be considered as that of the electric field Ex.

3- Field data

The applicability of the proposed technique has been tested on field data, A VLF-Resistivity survey has been conducted in the test site of the CRG, Garchy, France. The instrument used is the one manufactured by the CRG (Tabbagh et *al*., 1991). This test site (fig. 9) contains a wall with dimensions 10mx1mx1 meters buried in ground at a depth of 20 cm. The transmitter station used during this survey is the French Le Blanc with a frequency of 18 kHz.

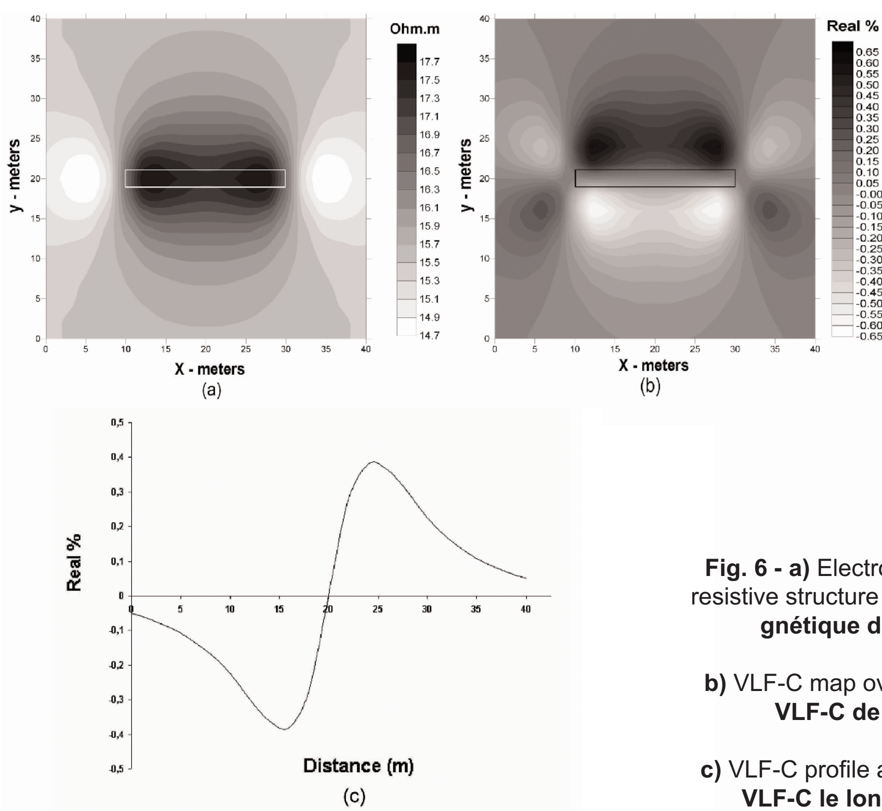

Fig. 6 - a) Electromagnetic-resistivity map over a resistive structure **(Carte de résistivité électromagnétique de la structure résistante)**.

b) VLF-C map over the resistant structure **(Carte VLF-C de la structure résistante)**.

c) VLF-C profile along Y-direction at X=20 **(Profil VLF-C le long de la direction Y à X=20)**.

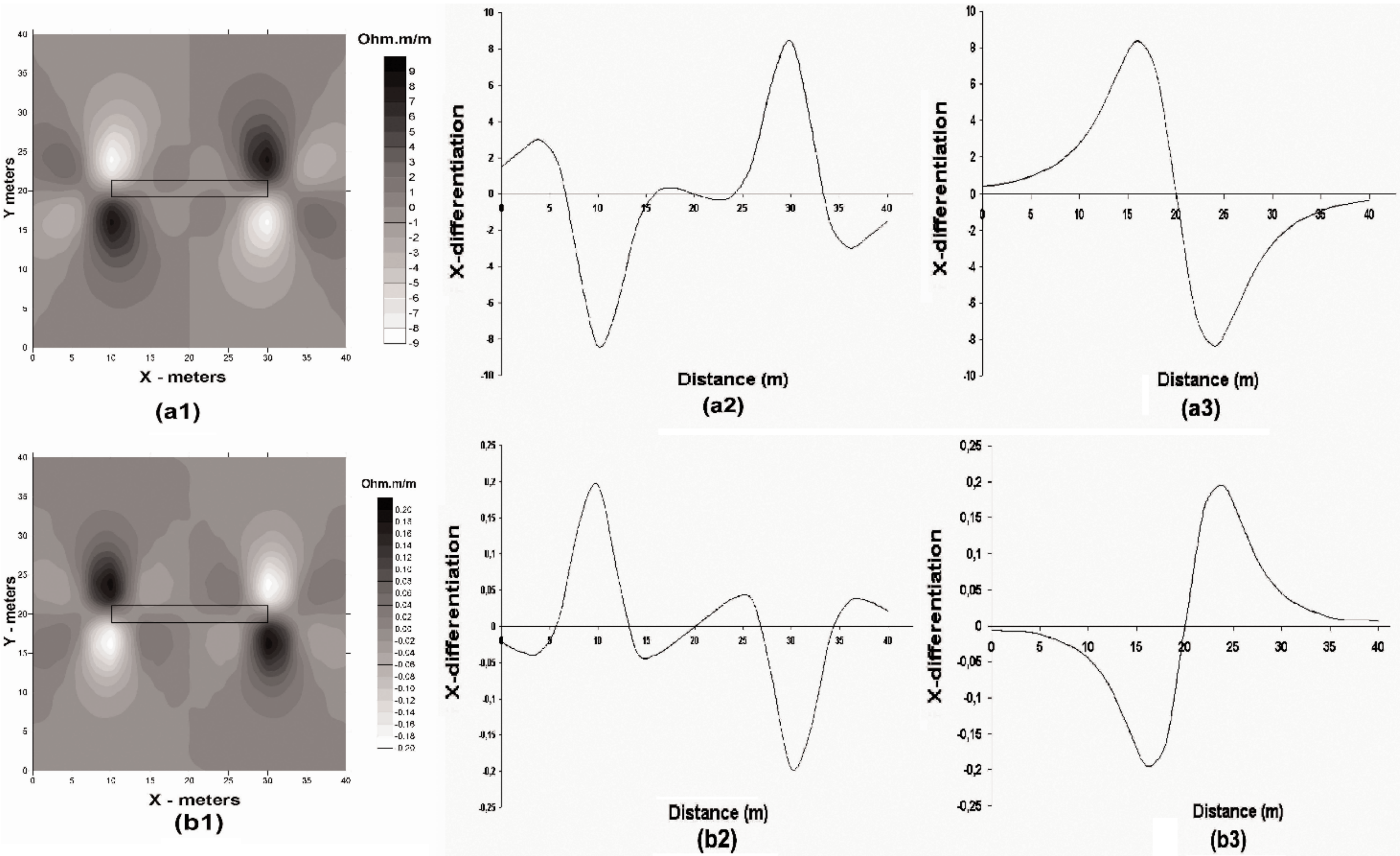

Fig. 7 - a1 and b1 : The X-derivative of the VLF-C map showing the four extrema ***(La dérivée suivant la direction X de la carte VLF-C montrant les quatre extrema)***.

a2 and b2 : Profiles along X-direction at Y=24 showing the extensions of the structures ***(Profils selon la direction X à Y=24 montrant les extensions des structures)***.

a3 and b3 : Profiles along Y-direction at X=10 showing the anomaly that is used to calculate the depth to top of the structure ***(Profils selon la direction Y à X=10 montrant l'anomalie utilisée pour le calcul de la profondeur du toit de la structure)***.

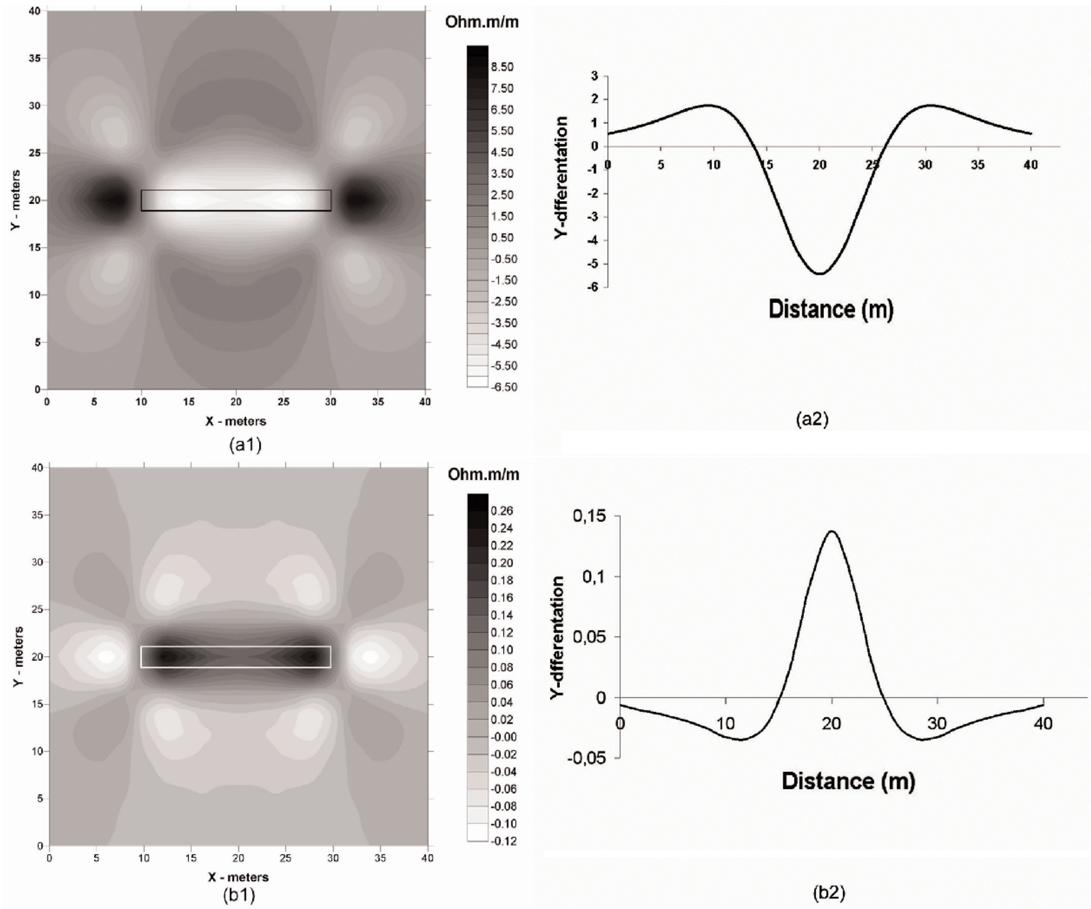

Fig. 8 - a1 and b1 : The Y-derivative of the VLF-C map showing the three extrema ***(La dérivée suivant la direction Y de la carte VLF-C montrant les trois extrema)***.

a2 and b2 : Profiles along Y-direction at X=20 showing the extensions of the structures ***(Profils selon la direction Y à X=20 montrant les extensions des structures)***.

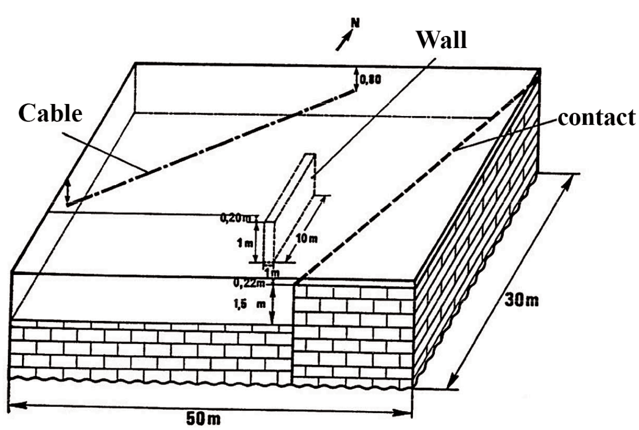

Fig. 9 - A buried wall of dimensions 10m x 1m x 10m, Garchy, France.

Mur enfoui de dimension 10m x 1m x 10m, Garchy, France.

The measured resistivity map is given in figure 10a, in which high resistivity values can be noticed in the middle of the map in the vicinity of the buried wall. The elongated form of the anomaly reflects the orientation of the wall.

In applying the proposed filter to the resistivity measurements, a VLF-C map (fig.10b) is produced showing a very clear typical VLF-EM anomaly over the wall. At the same time, the X- and Y-derivatives of the VLF-C give the expected results (figs. 10c and 10d), where the calculated dimensions of the wall are found to be 80 cm along the X-axis and 10 meters along the Y-axis. These calculated dimensions are in total coincidence with the field ones. However, the depth to top of the wall (in reality 20 cm) could not be determined, because any depth between 0m and 2m will always be given as 2m on the known standard curves normally used to calculate the depth.

CONCLUSION

As it is sometimes difficult to use the VLF-EM method in locating non-conducting structures, or it is difficult to use because of a weak signal or small ground resistivity changes, the use of the VLF-Resistivity method becomes very convenient. But the interpretation of the VLF-R map has stayed limited to a qualitative one. The proposed filter presented in this paper

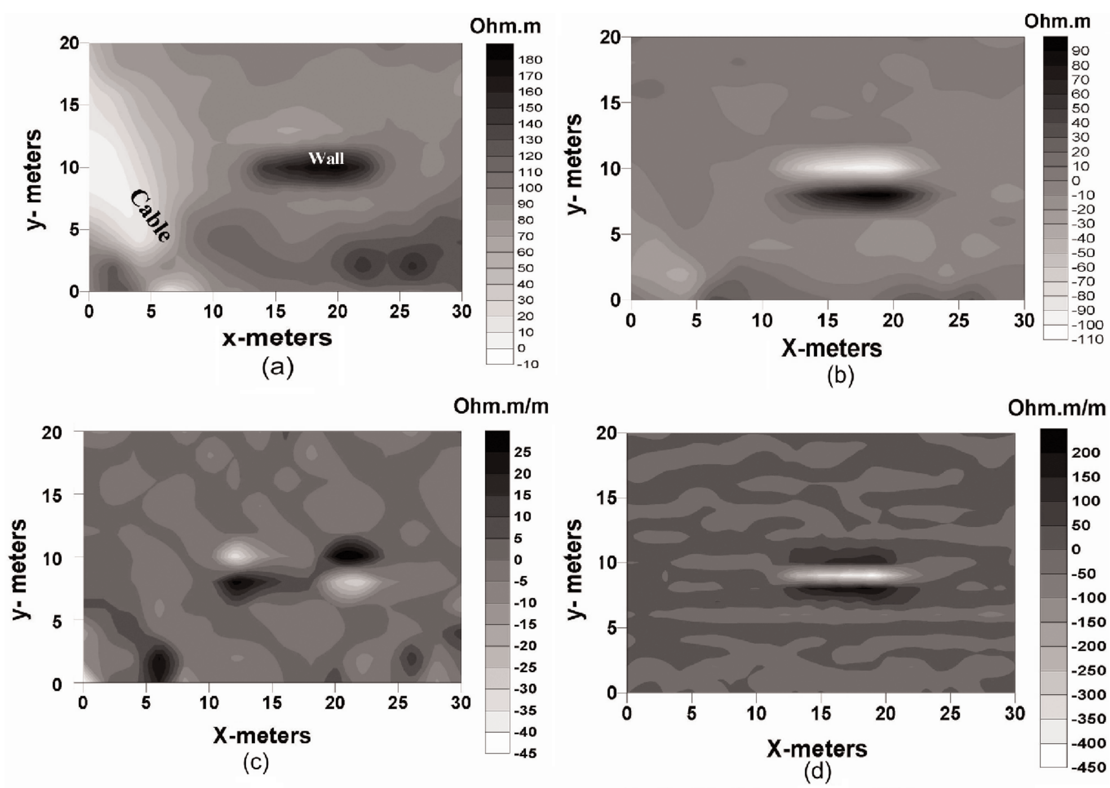

Fig. 10 - a : The measured electromagnetic-resistivity over the wall, Garchy, France ***(Résistivité électromagnétique mesurée au dessus du mur, Garchy, France)***.

b : The VLF-C map over the wall ***(Carte VLF-C au dessus du mur)***.

c : The X-derivative of the VLF-C map ***(La dérivée suivant X de la carte VLF-C)***.

d : The Y-derivative of the VLF-C map ***(La dérivée suivant Y de la carte VLF-C)***.

offers the opportunity to obtain more quantitative information of the causative structure, especially in shallow geophysical investigations (e.g. Civil engineering, archaeology, etc.).

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